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Total No. of Questions : 09

B.Tech.(ME)(2011 Onwards) (Sem.–5)

MATHEMATICS-III

Subject Code : BTAM-500

Paper ID : [A2127]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students has to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students has to attempt any TWO questions.

SECTION-A

1. **Write briefly :**

(a) Write down the Euler's formulae for the function: $f(x) = x^2, -c < x < c$.

(b) Define Laplace transformation of a function $f(t)$. State sufficient conditions for its existence.

(c) Find the Laplace transform of the function: $f(t) = \sin t\, u(t - \pi)$.

(d) Find all the regular and singular points of a differential equation :
$(1 - x^2)y'' - 2xy' + 12y = 0$.

(e) Prove that : $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

(f) From a p. d. e. form the equation :

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

(g) Solve : $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

(h) Prove that the function $f(z) = z$ is continuous but not differentiable at $z = 0$.

(i) Define conformal mapping with a suitable example. Also give an example of a non-conformal mapping.

(j) Find all the bilinear transformations whose fixed points are –1 and 1.

SECTION-B

2. Find the Fourier series to represent the function $f(x)=\begin{cases}\pi+x, & -\pi<x<0,\\ \pi-x, & 0<x<\pi.\end{cases}$

3. Apply Convolution Theorem to evaluate : $L^{-1}\left\{\frac{s^2}{\left(s^2+4\right)^2}\right\}$

4. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx=\frac{2}{2n+1}, if\ m\neq n.$

5. Solve the p.d.e. $x(y-z)p + y(z-x)q = z(x-y)$.

6. Prove that an analytic function with constant modulus is constant.

SECTION-C

7. (a) Evaluate : $\int_0^\infty t^3 e^{-t}\sin t\,dt$.

(b) Solve the differential equation by transform method :
$(D^2-1)x = a\cosh t,\ x(0)=x'(0)=0$.

8. (a) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, which satisfies the conditions :

$u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi x}{l}$.

(b) Prove that $\int J_3(x)dx=-J_2(x)-\frac{2}{x}J_1(x)+c$.

9. (a) Expand $f(z)=\frac{z}{(z-1)(2-z)}$ in Laurent series valid for

(i) $|z-1|>1$, (ii) $0<|z-2|<1$

(b) Evaluate : $\int_0^\pi \frac{d\theta}{3+\sin^2\theta}$.